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Do precision electroweak constraints guarantee e^+e^- collider discovery of at least one Higgs boson of a two-Higgs-doublet model?

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Abstract

We consider a CP-conserving two-Higgs-doublet type II model with a light scalar or pseudoscalar neutral Higgs boson ($h = h^0$ or $h = A^0$) that has no ZZ/WW coupling and, thus, cannot be detected in $e^+e^- \to Zh$ (Higgs-strahlung) or $\nu\overline{\nu}h$ (via WW fusion). Despite sum rules which ensure that the light h must have significant $t\overline{t}$ or $b\overline{b}$ coupling, for a wedge of moderate $\tan\beta$, that becomes increasingly large as m_h increases, the h can also escape discovery in both $b\overline{b}h$ and $t\overline{t}h$ production at a $\sqrt{s} = 500 - 800\,\mathrm{GeV}$ e^+e^- collider (for expected luminosities). If the other Higgs bosons happen to be too heavy to be produced, then no Higgs boson would be detected. We demonstrate that, despite such high masses for the other Higgs bosons, only the low-$\tan\beta$ portion of the no-discovery wedges in $[m_h, \tan\beta]$ parameter space can be excluded due to failure to fit precision electroweak observables. In the $\tan\beta \gtrsim 1$ regions of the no-discovery wedges, we find that the 2HDM fit to precision electroweak observables has small $\Delta\chi^2$ relative to the best minimal one-doublet SM fit.

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1. Introduction. Spontaneous gauge symmetry breaking in the Standard Model (SM) is realized by introducing a single Higgs doublet field, leading to a single CP-even Higgs boson, $h_{\rm SM}$. In this 1HDM, Higgs discovery at an e^+e^- collider is always possible via the Higgs-strahlung process, $e^+e^- \to Zh_{\rm SM}$ [1] provided only that $\sqrt{s} > m_Z + m_{h_{\rm SM}}$. However, even the simplest CP-conserving two-Higgs-doublet model (2HDM) extension of the SM exhibits a rich Higgs sector structure that makes Higgs discovery more challenging. The CP-conserving 2HDM predicts [2] the existence of two neutral CP-even Higgs bosons (h^0 and H^0, with $m_{h^0} \leq m_{H^0}$ by convention), one neutral CP-odd Higgs (A^0) and a charged Higgs pair ($H^\pm$). We consider the type-II 2HDM, wherein ϕ_1^0 couples to down-type quarks and leptons and ϕ_2^0 to up-type quarks. We employ the conventions $\langle\phi_{1,2}^0\rangle = v_{1,2} > 0$, $\tan\beta = v_2/v_1$. Tree-level Higgs coupling strengths relative to SM-like couplings are then ($s_\alpha \equiv \sin\alpha, \ldots$): $[t\overline{t}]$ $H^0, h^0, A^0 \to s_\alpha/s_\beta, c_\alpha/s_\beta, c_\beta/s_\beta$; $[b\overline{b}]$ $H^0, h^0, A^0 \to c_\alpha/c_\beta, -s_\alpha/c_\beta, s_\beta/c_\beta$; $[WW, ZZ]$ $H^0, h^0, A^0 \to \cos(\alpha-\beta), \sin(\beta-\alpha), 0$, where the $A^0 f\overline{f}$ couplings have an extra $i\gamma_5$ and the range $-\pi < \alpha \leq 0$ covers the full physical parameter space in the $m_{h^0} < m_{H^0}$ convention. [♯1]

In this note, we wish to address the extent to which a light h with zero ZZ/WW coupling (and, therefore, undetectable in the Zh and $\nu\overline{\nu}h$ final states) is guaranteed to be discovered in the $t\overline{t}h$ and/or $b\overline{b}h$ final states at an e^+e^- LC collider even if all other Higgs bosons are too heavy to be produced *and* if the model is required to be consistent with precision electroweak constraints. We consider the cases of: a) $h = A^0$ (the tree-level A^0ZZ and A^0WW couplings are automatically zero); and b) $h = h^0$ with the scalar sector mixing angle chosen so that $\sin(\beta-\alpha) = 0$ (to zero the h^0ZZ and h^0WW couplings).[♯2] In earlier work [3], we found that there are dangerous wedge-shaped regions in the m_h-$\tan\beta$ plane, characterized by a range of moderate $\tan\beta$ values that increases in size as m_h increases, for which the h cannot be seen at a future LC, even though it is guaranteed that the $t\overline{t}h$ and $b\overline{b}h$ couplings-squared cannot be simultaneously suppressed when the ZZh coupling is zero. Even for a very high integrated luminosity, $L = 2500~\mathrm{fb}^{-1}$, such as might be achievable after several years of running, at both $\sqrt{s} = 500\,\mathrm{GeV}$ and $800\,\mathrm{GeV}$ and m_h well below $\sqrt{s} - 2m_t$, the $t\overline{t}h$ and $b\overline{b}h$ production rates are simply not adequate for detecting the h. However, since the other Higgs bosons are assumed to be quite heavy to avoid production, implying no *light* Higgs with substantial ZZ/WW couplings, it would seem that the fit to precision electroweak constraints is likely to be poor in the no-discovery wedges. But, in [4] it was shown that a good global fit to EW data is possible even for very light h^0 or A^0. In this paper, we compute

[♯1] We employ the conventional definition of α according to which $H^0 = \cos\alpha\sqrt{2}\Re\phi_1^0 + \sin\alpha\sqrt{2}\Re\phi_2^0$ and $h^0 = -\sin\alpha\sqrt{2}\Re\phi_1^0 + \cos\alpha\sqrt{2}\Re\phi_2^0$. The $0 < \alpha \leq \pi$ range is equivalent to $-\pi < \alpha \leq 0$ under $\alpha \to \alpha + \pi$ which is equivalent to simultaneously redefining the overall signs of the h^0 and H^0 states; this has no physical consequences.

[♯2] We have checked that the non-zero ZZ/WW couplings induced at one-loop are too small to produce a detectable Zh or $\nu\overline{\nu}h$ signal. E.g., relative to the Zh and $\nu\overline{\nu}h$ cross sections for a SM-like h, the suppressions for $h = A^0$ are of order $\alpha_W^2 \frac{m_t^4}{s m_Z^2} \cot^2\beta$ and $\alpha_W^2 \cot^2\beta$, respectively.

the best precision electroweak fit χ^2's that can be achieved in the no-discovery wedges. Generally speaking, for LC $\sqrt{s} = 500\,\mathrm{GeV}$ ($800\,\mathrm{GeV}$) we find that the $\tan\beta \gtrsim 2$ portions of the 2HDM no-discovery wedges in m_h-$\tan\beta$ parameter space have $\Delta\chi^2 < 1$ (< 1.5) (relative to the best SM fit) and all of the no-discovery wedges' portions with $\tan\beta \gtrsim 1$ have $\Delta\chi^2 < 2$. The discrimination (following from current precision electroweak data) between the minimal 1HDM SM and the no-discovery scenarios in the 2HDM is thus rather weak for $\sqrt{s} = 500\,\mathrm{GeV}$ and not all that much better for $\sqrt{s} = 800\,\mathrm{GeV}$. After presenting the results for the EW precision fits and qualitative discussion we find an interestingly simple form of the Higgs potential which ensures the smallest $\Delta\chi^2$'s in the scenarios considered. Finally, we comment on extending the energy of the LC and searches at the LHC.

2. No-discovery wedges. We begin with a reminder and discussion concerning the wedges of m_h-$\tan\beta$ parameter space for which the h cannot be discovered. Our criterion is that there should be fewer than 50 events in $t\bar{t}h$ and in $b\bar{b}h$ production assuming $L = 2500\ \mathrm{fb}^{-1}$ and $\sqrt{s} = 500$ or $800\,\mathrm{GeV}$. (In practice, larger event numbers might turn out to be needed for discovery; 50 events before cuts is probably a minimum.) The wedges are shown by the solid black lines in Fig. 1 for $h = A^0$ and in Fig. 2 for $h = h^0$. These figures are confined to m_h values such that $t\bar{t}h$ production is kinematically allowed for the given $\sqrt{s}$. The no-discovery zones extend beyond $m_h = \sqrt{s} - 2m_t$ with the difference that there is no lower-bound on $\tan\beta$. However, low $\tan\beta$ values are highly disfavored by the precision electroweak constraints for all m_h. We shall discuss the case of $m_h \sim \sqrt{s}$ later. The $h = A^0$ wedges are essentially the same as those found for the general CP-violating 2HDM in [3]. Even though the A^0 has exactly the same magnitude (but not necessarily the same sign) for its $\overline{f} i\gamma_5 f$ couplings as does the h^0 for its $\overline{f} 1 f$ couplings [for $\sin(\beta - \alpha) = 0$], the low (high) $\tan\beta$ limits of the $h = h^0$ wedges are substantially above (close to) those for $h = A^0$. This is because the $f\overline{f}h^0$ cross section differs from the $f\overline{f}A^0$ cross section by a term proportional to m_f^2; the resulting difference is substantial for $f = t$ (which determines the low $\tan\beta$ limit of the wedge).

3. Electroweak fits. The question is the extent to which portions or all of these wedges can be excluded because the model is inconsistent with precision electroweak constraints when the masses of the other Higgs bosons are taken to be sufficiently heavy that they cannot be produced. For the heavy neutral (charged) Higgs bosons, the strongest constraint of this type is that $b\bar{b}H$ ($tbH^\pm$) production be forbidden. [♯3] Figs. 1 and 2 show the best $\Delta\chi^2$ values obtained for the precision electroweak fit at each $\tan\beta$ and m_h ($= m_{A^0}$ or m_{h^0}) value in the $\sqrt{s} = 500$ and $800\,\mathrm{GeV}$ no-discovery wedges after scanning over all values of masses for the other (heavier) neutral Higgs bosons above $\sqrt{s} - 10\,\mathrm{GeV}$, and over all $H^\pm$ masses above $\sqrt{s} - 180\,\mathrm{GeV}$. In the case of $h = A^0$, we also scan over all mixing angles α in the CP-even sector. The observables we consider are listed in Table 1. We employ the LEP/SLD/Tevatron experimental values from [6] with the exception of m_W^{LEP} for

[♯3] If $b\bar{b}H$ is forbidden, $\nu\overline{\nu}H$ will be extremely suppressed.

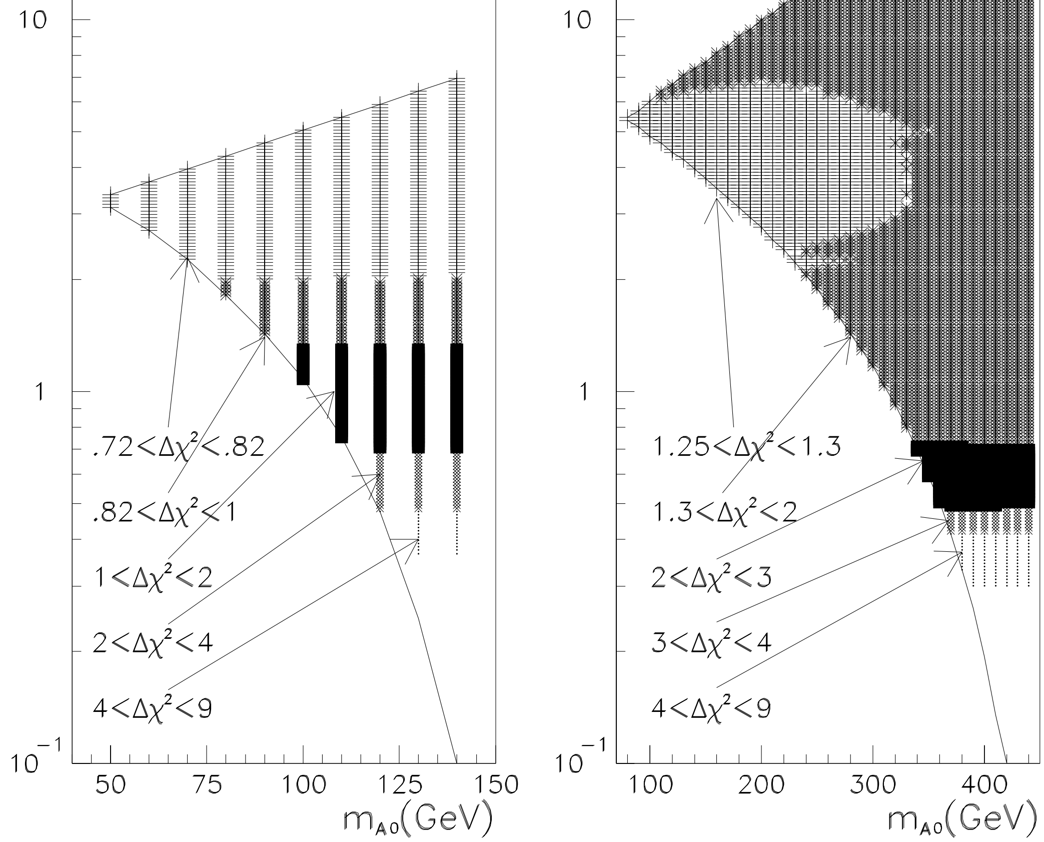

Figure 1: For $\sqrt{s} = 500\,\mathrm{GeV}$ and $\sqrt{s} = 800\,\mathrm{GeV}$, the solid lines show as a function of m_{A^0} the maximum and minimum $\tan\beta$ values between which $t\bar{t}A^0$, $b\bar{b}A^0$ final states will both have fewer than 50 events assuming $L = 2500\ \mathrm{fb}^{-1}$. The different types of bars indicate the best χ^2 values obtained for fits to precision electroweak data after scanning: over the masses of the remaining Higgs bosons subject to the constraint they are too heavy to be directly produced; and over the mixing angle in the CP-even sector.

which we take the LEP2 result [7]. We include statistical and systematic errors and correlations in computing the contribution to χ^2 from each observable. We observe that rather small $\Delta\chi^2$ levels are the rule once $\tan\beta \gtrsim 1$; $\Delta\chi^2 < 1$ (< 2) is easily achieved in the $\sqrt{s} = 500\,\mathrm{GeV}$ ($800\,\mathrm{GeV}$) wedges. For the no-discovery parameter regions, the best χ^2 values for $\tan\beta > 0.5$ are always obtained in the $h = A^0$ (h^0) cases, respectively, by taking the next lightest Higgs boson H to be $H = h^0$ (H^0) with m_{h^0} (m_{H^0}) as light as consistent with forbidding $b\bar{b}H$ production (i.e., $m_H = \sqrt{s} - 10\,\mathrm{GeV}$). The large $\Delta\chi^2$'s found for $\tan\beta$ values substantially below 1 derive primarily from too large a result for R_b, although the deviation of Γ^Z_{tot} from the observed value also becomes increasingly large as $\tan\beta$ becomes small.

Given that low $\tan\beta$ is disfavored by the precision electroweak fit, it is appropriate to explore the $\tan\beta \gtrsim 0.5$ region out to values of m_h beyond the $m_h < \sqrt{s} - 2m_t$

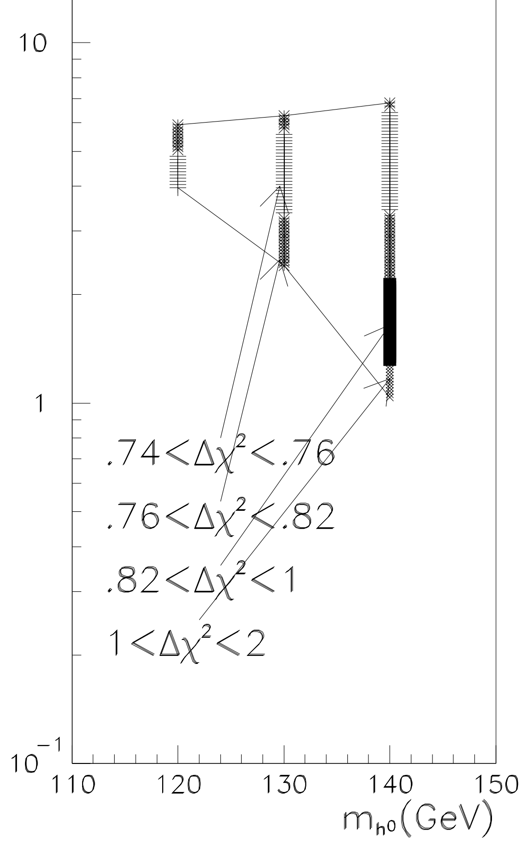

Figure 2: The same as for Fig. 1, except for $h = h^0$. The CP-even sector mixing angle is fixed by the requirement $\sin(\beta - \alpha) = 0$.

no-discovery wedges considered originally in [3]. The figures make clear that the no-discovery region for the h (whether A^0 or h^0) reaches to increasingly higher $\tan\beta$ as m_h increases. To illustrate, we give additional results for $m_h = \sqrt{s} - 10\,\mathrm{GeV}$, the smallest m_h for which $b\bar{b}$+Higgs is forbidden for all neutral Higgs bosons. The boundaries in $\tan\beta$ for different $\Delta\chi^2_{\rm min}$ levels are given in Table 2.

To illustrate some additional points, we focus on the $h = A^0$ case, which also allows us to discuss issues related to choosing the scalar-sector mixing angle α. Naively, it might be expected that the smallest $\Delta\chi^2$ would be achieved when $H = h^0$ has SM-like couplings to ZZ, WW ($\beta - \alpha \sim \pi/2$). This is frequently the case, and, in such cases, $m_{H^\pm}$ and m_{H^0} are typically much larger than m_{h^0}, while $m_{H^\pm}$ is closely correlated with and just slightly larger than m_{H^0}. However, another frequent $\Delta\chi^2_{\rm min}$ configuration is $\alpha \sim 0$ (i.e. no mixing in the scalar sector). For such cases, $m_{H^\pm} - m_{H^0}$ is again not large and, in addition, $m_{H^0} - m_{h^0}$ is often small (sometimes $m_{H^0} = m_{h^0}$, in which case changing α has no physical consequences). Further, $\Delta\chi^2_{\rm min}$ cases with $\alpha \sim 0$ typically arise for $\tan\beta$ values for which $\sin^2(\beta - \alpha)$ is rather small (< 0.2). However, it is very common that the smallest $\Delta\chi^2$ achieved with $\sin^2(\beta - \alpha) \sim 1$ is only slightly larger than that found

Table 1: Observables considered (TEV stands for Tevatron data) and typical pulls for a 2HDM fit. Pulls are defined as $(\mathcal{O}_i - \mathcal{O}_i^{\rm min})/\Delta\mathcal{O}_i$, where $\mathcal{O}_i$ is the measured value of a given observable, $\mathcal{O}_i^{\rm min}$ is the value for the observable for the best fit choice of parameters, and $\Delta\mathcal{O}_i$ is the full error (including systematic error) for that observable. The pull results are for $m_t = 174\,{\rm GeV}$, $\alpha_s = 0.117$, $m_{A^0} = 90\,{\rm GeV}$, $\tan\beta = 2.3$, $m_{h^0} = 490\,{\rm GeV}$, $m_{H^0} = 830\,{\rm GeV}$ and $m_{H^\pm} = 850\,{\rm GeV}$, yielding $\Delta\chi^2 = 0.78$ relative to the best χ^2 achieved in the SM-like limit of the 2HDM, for which we also give the pulls for the same m_t and α_s. These latter results are quite close to those given in [6] with the exception of $m_W^{\rm LEP}$ for which we have used the Moriond result including LEP2 running [7]. The SM-like 2HDM pulls are essentially identical to those of [6] if we use $m_W^{\rm LEP}$ as quoted there.

$\mathcal{O}$	$m_W^{\rm LEP}$	$m_W^{\rm TEV}$	$\sin^2\theta_W^{\rm TEV}$	$\Gamma^Z_{\rm tot}$	$\sigma^Z_{\rm had}$	$\mathcal{A}_e^{\rm LEP}$
2HDM	0.157	0.880	1.32	-0.972	1.61	0.338
SM	0.370	1.04	1.23	-0.508	1.73	0.167
$\mathcal{O}$	$\mathcal{A}_\tau^{\rm LEP}$	$\sin^2\theta^*_{\rm LEP}$	$\Gamma^Z_{\rm had}/\Gamma^Z_{\rm lep}$	$A_{FB}^{l\ \rm LEP}$	$R_b^{\rm LEP}$	$R_c^{\rm LEP}$
2HDM	-0.927	0.522	1.42	0.944	0.733	-0.744
SM	-1.12	0.632	1.13	0.742	0.668	-0.743
$\mathcal{O}$	$A_{FB}^{b\ \rm LEP}$	$A_{FB}^{c\ \rm LEP}$	$A_{LR}^{b\ \rm SLD}$	$A_{LR}^{c\ \rm SLD}$	$\sin^2\theta_{\rm SLD}$	
2HDM	-1.98	-1.22	-0.948	-1.45	-2.26	
SM	-2.29	-1.34	-0.950	-1.46	-1.83	

for $\alpha \sim 0$, and vice versa. Indeed, for most $[m_{A^0}, \tan\beta]$ points in the no-discovery wedges, $\Delta\chi^2$ values quite close to $\Delta\chi^2_{\rm min}$ can be achieved for a rather wide range of 2HDM parameters so long as $m_{H^\pm} - m_{H^0}$ is positive but not large.

To illustrate in more detail, consider $\sqrt{s} = 500\,{\rm GeV}$ and the no-discovery point $[m_{A^0} = 90\,{\rm GeV}, \tan\beta = 2.3]$, for which $\Delta\chi^2_{\rm min}$ is achieved for $m_{h^0} = 490\,{\rm GeV}$ (i.e., as small as consistent with absence of $b\bar{b}h^0$ production), $m_{H^0} = 830\,{\rm GeV}$, $m_{H^\pm} = 850\,{\rm GeV}$ and $\alpha = -0.1\pi$ (for which $\beta - \alpha \sim \pi/2$, implying $\sin^2(\beta-\alpha) \sim 1$). The 'pulls' of each of our 17 input observables, $\mathcal{O}_i$, for this $\Delta\chi^2_{\rm min}$ parameter set are listed in Table 1. These values are typical. From the table, we also see that the 2HDM pattern of pulls is only somewhat different from the conventional 1HDM fit pulls obtained using the same input data, with some $\mathcal{O}_i$ fit better in the 2HDM case than in the 1HDM case and vice versa.

For $m_{h^0} = 490\,{\rm GeV}$, a large range of values for m_{H^0} and $m_{H^\pm}$ is possible without increasing $\Delta\chi^2$ significantly, provided $m_{H^\pm}$ is closely correlated with m_{H^0}. This is illustrated in Fig. 3 which shows the values for $m_{H^\pm}$ as a function of m_{H^0} for $\Delta\chi^2 < 0.8$ and in the ranges [0.8,1], [1,2], and [2,3]. For the smallest $\Delta\chi^2$ values, $m_{H^\pm} - m_{H^0} = 20\,{\rm GeV}$.

Not surprisingly, there is an increase of minimum $\Delta\chi^2$ as m_{h^0} increases (for given m_{A^0} and $\tan\beta$). Also, the best choices of m_{H^0} and $m_{H^\pm}$ are much larger than m_{h^0} and they increase as m_{h^0} is increased. Fig. 4 illustrates this for the case

Table 2: Lower and upper values of $\tan\beta$, using the notation $[\tan\beta_{\rm min}, \tan\beta_{\rm max}]$, at which the given $\Delta\chi^2_{\rm min}$ value is crossed for the $m_h = \sqrt{s} - 10\,{\rm GeV}$ cases.

$\Delta\chi^2_{\rm min}$	1	2	3	4	9
$h = A^0, \sqrt{s} = 500$	[1.8,14]	[0.63,56]	[0.49,75]	[0.44,89]	[0.30, $>$ 110]
$h = A^0, \sqrt{s} = 800$	no	[0.75,47]	[0.46,85]	[0.39,107]	[0.27, $>$ 110]
$h = h^0, \sqrt{s} = 500$	no	[0.92,51]	[0.73,73]	[0.63,86]	[0.45, $>$ 110]
$h = h^0, \sqrt{s} = 800$	no	[1.4,33]	[0.68,78]	[0.55,102]	[0.35, $>$ 110]

$[m_{A^0} = 90\,{\rm GeV}, \tan\beta = 2.3]$. For all m_{h^0}, $\Delta\chi^2_{\rm min}$ is achieved for $\beta - \alpha \sim \pi/2$, corresponding to maximal coupling of the lightest scalar h^0 to ZZ. However, as to be expected from Fig. 3, if we look for all solutions with $\Delta\chi^2 < \Delta\chi^2_{\rm min} + 0.05$, many more choices of closely correlated $m_{H^\pm}, m_{H^0}$ pairings are possible with much lower overall mass scale and these do not always have $\beta - \alpha \sim \pi/2$. Fig. 4 also shows that if we decrease the minimum value allowed for m_{h^0} to $\sqrt{s} - m_Z = 410\,{\rm GeV}$ (the value for which Zh^0 production is forbidden when $\sqrt{s} = 500\,{\rm GeV}$), then $\Delta\chi^2_{\rm min}$ decreases from ~ 0.86 to ~ 0.71. Since the $b\bar{b}h^0$ cross section is very small for $m_{h^0} = 410\,{\rm GeV}$ and $\tan\beta = 2.3$ and $\nu\bar{\nu}h^0$ is very slow to turn on, this would still be consistent with non-observation of the h^0.

In the case of $h = h^0$, the analogue of the above discussion applies except that we fix $\alpha = \beta$ (for zero ZZh and WWh couplings), implying that the next lightest Higgs, which is always $H = H^0$ for small $\Delta\chi^2$, is SM-like. One achieves $\Delta\chi^2_{\rm min}$ for $m_{H^0} = \sqrt{s} - 10\,{\rm GeV}$ and for small positive $m_{H^\pm} - m_{A^0}$ with m_{A^0} and $m_{H^\pm}$ typically substantially larger than m_{H^0}. Again, if we force m_{H^0} to larger values for any given $[m_{A^0}, \tan\beta]$ choice, then $\Delta\chi^2_{\rm min}$ increases as do the associated $m_{H^\pm}, m_{A^0}$ masses.

Finally, we note that sensitivity of $\Delta\chi^2_{\rm min}$ to precise inputs is small. For example, by switching from fixed m_b to running m_b in the Higgs loop graphs (a change that is part of the two-loop corrections), the minimum χ^2 values achievable in the SM-like limit and in the no-discovery scenarios both decrease by about 0.4 for the running mass choice, leaving $\Delta\chi^2_{\rm min}$ almost the same. Changes in input values of m_t, α_s and $m_W^{\rm LEP}$ also lead to compensating changes. We believe $\Delta\chi^2_{\rm min}$ to be stable against such changes/choices to within 0.1. More sensitive are the pulls and the precise heavy Higgs masses that yield $\chi^2_{\rm min}$. While small values for the splitting between the heaviest neutral and the charged Higgs masses are always preferred, the overall mass scale at $\chi^2_{\rm min}$ is quite sensitive to precise inputs.

4. Qualitative discussion. It is useful to understand in an approximate analytic fashion how it is that there need not be a light scalar Higgs boson with substantial ZZ coupling in order for predictions for all the precision observables to be in good agreement with data (see also [4]). Consider the quantity $\Delta\rho$ which

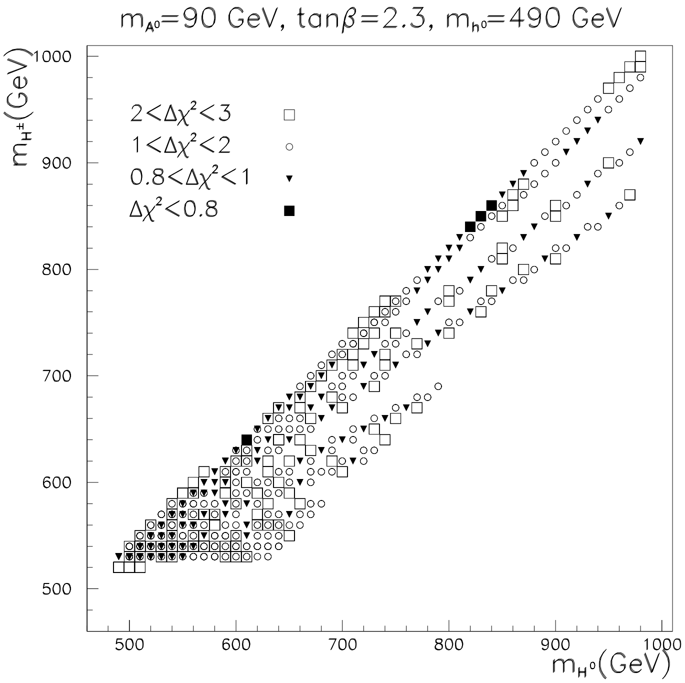

Figure 3: For the case of $m_{A^0} = 90\,\mathrm{GeV}$, $\tan\beta = 2.3$ and $m_{h^0} = 490\,\mathrm{GeV}$ (that yields the minimum $\Delta\chi^2$ for $\sqrt{s} = 500\,\mathrm{GeV}$ when requiring that $b\bar{b}h^0$ production is forbidden), we plot $m_{H^\pm}$ as a function of m_{H^0} for various ranges of $\Delta\chi^2$. Scans in m_{H^0} and $m_{H^\pm}$ were done using 10 GeV steps, which leads to some incompleteness in the points for each $\Delta\chi^2$ range. The scan in m_{H^0} was limited to $m_{H^0} < 980\,\mathrm{GeV}$. Multiple entries at the same $m_{H^0}, m_{H^\pm}$ location correspond to different α values.

must be very small for a low χ^2 fit [5]. It is convenient to write

$$\begin{aligned}\Delta\rho &= \frac{\alpha}{16\pi s_W^2 m_W^2}\Big(\cos^2(\beta-\alpha)[f(m_{A^0},m_{H^\pm})+f(m_{H^\pm},m_{h^0})-f(m_{A^0},m_{h^0})] \\ &+ \sin^2(\beta-\alpha)[f(m_{A^0},m_{H^\pm})+f(m_{H^\pm},m_{H^0})-f(m_{A^0},m_{H^0})]\Big) \\ &+ \cos^2(\beta-\alpha)\Delta\rho_{SM}(m_{H^0})+\sin^2(\beta-\alpha)\Delta\rho_{SM}(m_{h^0})\,, \end{aligned}\tag{1}$$

where

$$f(x,y) = \frac{x^2+y^2}{2} - \frac{x^2y^2}{x^2-y^2}\log\frac{x^2}{y^2}\tag{2}$$

is symmetric in $x \leftrightarrow y$, vanishes for $x = y$ and is > 0 for $x \neq y$, and

$$\Delta\rho_{SM}(m) = -\frac{3\alpha}{16\pi s_W^2 m_W^2}\,[f(m,m_W)-f(m,m_Z)] - \frac{\alpha}{8\pi c_W^2}\,.\tag{3}$$

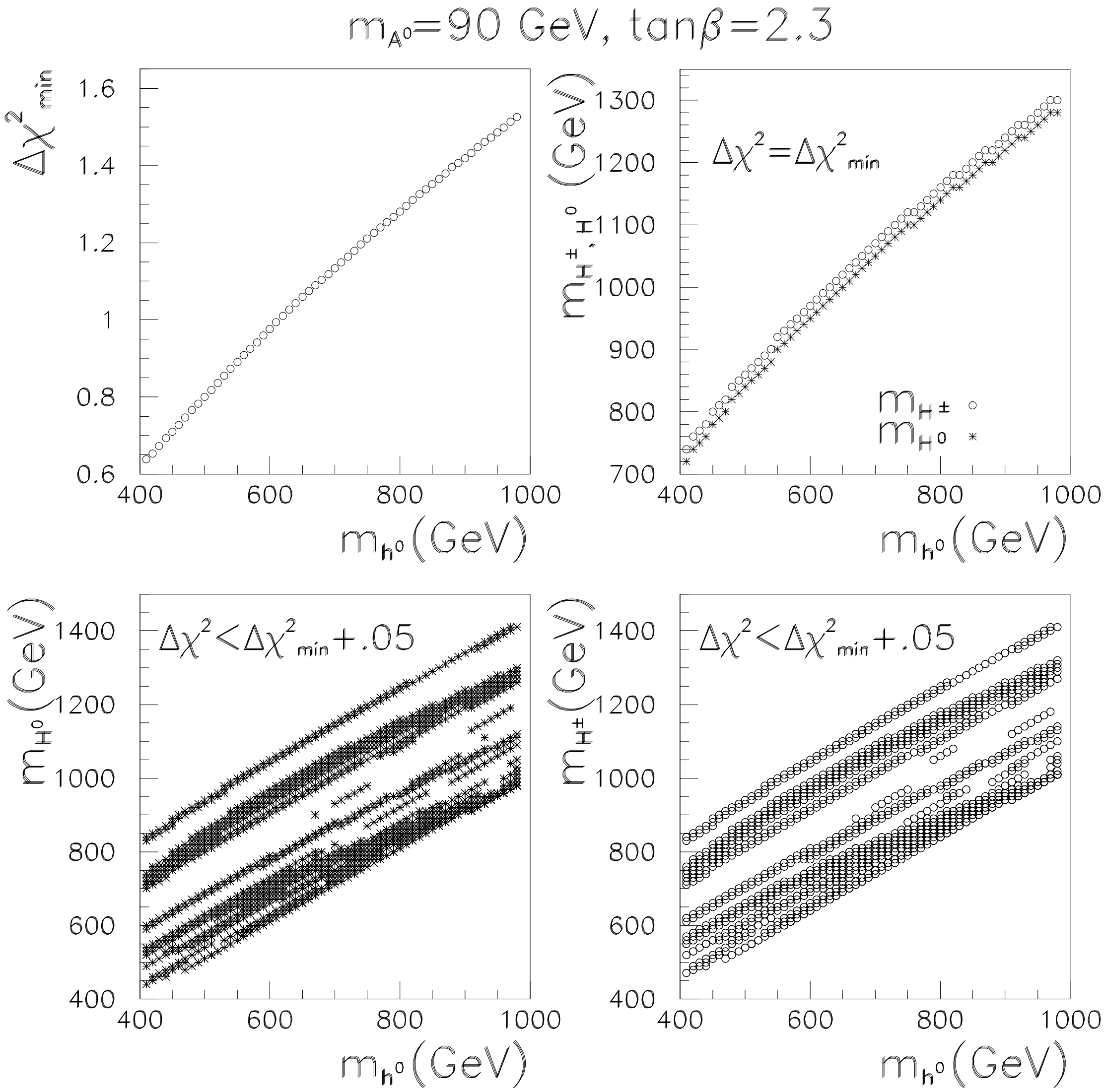

Figure 4: For the case of $m_{A^0} = 90\,\mathrm{GeV}$ and $\tan\beta = 2.3$, we plot as a function of $m_{h^0} \in [410, 980]\,\mathrm{GeV}$: a) the minimum $\Delta\chi^2$ found after scanning over all values of $m_{H^0}, m_{H^\pm} > m_{h^0}$ and over all scalar sector mixing angles; b) the corresponding values of m_{H^0} and $m_{H^\pm}$; c) the values of m_{H^0} for which $\Delta\chi^2 < \Delta\chi^2_{\min} + 0.05$ is achieved; d) the closely correlated values of $m_{H^\pm}$ for which $\Delta\chi^2 < \Delta\chi^2_{\min} + 0.05$ is achieved. For this case, $\Delta\chi^2_{\min}$ is always achieved for $\alpha = -0.1\pi$ (our scan is in units of 0.1π), which roughly corresponds to $\beta - \alpha = \pi/2$, i.e. maximal h^0 coupling to ZZ.

The usual SM-like result, $\Delta\rho = \Delta\rho_{SM}(m_{h^0})$, is obtained by taking $m_{A^0} = m_{H^0} = m_{H^\pm}$ and $\sin^2(\beta-\alpha) = 1$ (for which h^0 carries all the ZZ coupling).

Consider the case of $h = A^0$ and $H = h^0$, for which m_{h^0} must be large (to forbid $b\bar{b}h^0$ production) and $m_{H^\pm} - m_{H^0}$ should be small and positive for a good χ^2. For cases such that $\Delta\chi^2_{\rm min}$ is achieved with $\sin^2(\beta-\alpha)\sim 1$, a simple expression can be given for $\Delta\rho$. Using Eqs. (1) and (3), and expanding appropriately, we find

$$\Delta\rho = \frac{\alpha}{16\pi m_W^2 c_W^2}\left\{\frac{c_W^2}{s_W^2}\frac{m_{H^\pm}^2 - m_{H^0}^2}{2} - 3m_W^2\left[\log\frac{m_{h^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2}\log\frac{m_W^2}{m_Z^2}\right]\right\} \quad (4)$$

From this expression, we see that the (negative) logarithmic increase with increasing $m_{h^0}^2$ (that is the same as present in the 1HDM SM) can actually be canceled by a small (positive) difference between $m_{H^\pm}^2$ and $m_{H^0}^2$, which is precisely the nature of the $\Delta\chi^2_{\rm min}$ arrangements we find. It is apparent from this result that actual degeneracy, $m_{h^0}\simeq m_{H^0}\simeq m_{H^\pm} = M$, would lead to bad logarithmic growth of $\Delta\rho$ with M (as bad as in the SM). We also note that small $\Delta\rho$ does not require $\cos^2(\beta-\alpha)\sim 0$; cancellations similar to those illustrated above can still be arranged in the more general case, although they are more complicated. Finally, for the case of a light $h = h^0$, $\Delta\rho$ takes the form given above with $m_{H^0}^2 \to m_{A^0}^2$ and $m_{h^0}^2 \to m_{H^0}^2$, and corresponding cancellations can be arranged by having a small positive $m_{H^\pm}^2 - m_{A^0}^2$.

Another observable of interest is the S parameter. For light $h = A^0$ (h^0), small $m_{H^\pm}^2 - m_{H^0}^2$ ($m_{H^\pm}^2 - m_{A^0}^2$) is always needed for good χ^2 fits. In this limit, independent of the size of $m_{H^0}^2 - m_{h^0}^2$ ($m_{A^0}^2 - m_{H^0}^2$) (and independent of $\sin^2(\beta-\alpha)$ in the $h = A^0$ case to the extent that $m_{A^0}\sim m_W$) we find

$$S(0) \sim \frac{1}{12\pi}\left(-\frac{5}{3} + \log\frac{m_H^2}{m_W^2}\right), \quad (5)$$

where $H = h^0$ ($H = H^0$), respectively. The fact that S grows logarithmically implies that the χ^2 of the precision electroweak fit will slowly worsen with increasing mass m_H of the lightest kinematically inaccessible Higgs boson, as noted, for example, in discussing the $[m_{A^0} = 90\,{\rm GeV}, \tan\beta = 2.3]$ case. The non-decoupling effects in S (and other observables) of large m_H imply that as m_H is forced to increase with $\sqrt{s}$ (as required to avoid H discovery in the ZH channel if not the $b\bar{b}H$ channel) the $\Delta\chi^2_{\rm min}$ achievable will slowly increase relative to the SM-like limit of a light h^0 with full WW, ZZ coupling strength.

5. The Higgs potential. It is interesting to ask if the parameter configurations that minimize $\Delta\chi^2$ could find explanation in symmetries at the Lagrangian level. After imposing the usual Z_2 symmetry to avoid hard flavor changing terms in the Lagrangian, the 2HDM potential can be written in terms of the two SU(2) Higgs doublets $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$ in the form (see, e.g., [3]):

$$V(\Phi_1,\Phi_2) \;=\; -\mu_1^2\Phi_1^\dagger\Phi_1 - \mu_2^2\Phi_2^\dagger\Phi_2 - (\mu_{12}^2\Phi_1^\dagger\Phi_2 + {\rm h.c.}) + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2$$

$$+\quad \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4|\Phi_1^\dagger\Phi_2|^2+\frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2+\text{h.c.}\right]\,, \tag{6}$$

where μ_{12}^2 and λ_5 should be chosen real for a CP-conserving Higgs potential. The resulting Higgs masses or mass matrices are then

$$\begin{aligned} m_{A^0}^2 &= \frac{\mu_{12}^2}{s_\beta c_\beta}-v^2\lambda_5\,, \quad m_{H^\pm}^2=m_{A^0}^2+\frac{1}{2}v^2(\lambda_5-\lambda_4) \\ \mathcal{M}^2 &= m_{A^0}^2\begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2\end{pmatrix}+v^2\begin{pmatrix}\lambda_1 c_\beta^2+\lambda_5 s_\beta^2 & (\lambda_3+\lambda_4)s_\beta c_\beta \\ (\lambda_3+\lambda_4)s_\beta c_\beta & \lambda_2 s_\beta^2+\lambda_5 c_\beta^2\end{pmatrix}. \end{aligned} \tag{7}$$

So long as $m_{A^0}^2>0$, the CP-conserving minimum is either the only minimum ($\lambda_5>0$) or the preferred minimum ($\lambda_5<0$). Let us look first for relations among the λ's that will lead to the $m_{H^\pm}\gtrsim m_{H^0}>m_{h^0}\gg m_{A^0}$ and $\beta-\alpha\sim\pi/2$ configuration. Consider the case where $\lambda_1=\lambda_2$. To obtain $\beta-\alpha=\pi/2$ we must, first of all, have $\tan 2\alpha=\tan 2\beta$, which requires $\lambda_1-\lambda_5=\lambda_3+\lambda_4$. Then, $m_{h^0}^2=\lambda_1 v^2$ and $m_{H^0}^2=m_{A^0}^2+\lambda_5 v^2$. In the limit of small m_{A^0}, we then require $\lambda_1<\lambda_5$ for $m_{h^0}<m_{H^0}$. Further, for $m_{H^\pm}\sim m_{H^0}$ when m_{A^0} is small, we need $\lambda_4=-\lambda_5$. Then, $\lambda_1-\lambda_5=\lambda_3+\lambda_4$ implies $\lambda_1=\lambda_3$. Meanwhile, $m_{A^0}^2$ will be small if $\mu_{12}^2\sim\lambda_5 s_\beta c_\beta v^2$. Thus, our potential should have

$$\lambda_1\sim\lambda_2\sim\lambda_3>0\,,\quad \lambda_5=-\lambda_4>0\,,\quad \lambda_5>\lambda_1\,. \tag{8}$$

This configuration is achieved for a rather amusing special form of the quartic terms in the potential:

$$V_{\text{quartic}}(\Phi_1,\Phi_2)=\frac{1}{2}\lambda_1\left|\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right|^2-\frac{1}{2}\lambda_5\left|\Phi_1^\dagger\Phi_2-\Phi_2^\dagger\Phi_1\right|^2\,, \tag{9}$$

i.e. a weighted sum of the (absolute) squares of the natural symmetric and anti-symmetric combinations of the two Higgs doublet fields. This form of the potential guarantees absence of quadratic growth of $\Delta\rho$ with the masses of the heavier Higgs bosons, i.e. it incorporates a hidden custodial SU(2) symmetry. We need only break these various relations by relatively small amounts in order to achieve the small $m_{H^\pm}-m_{H^0}$ mass difference required for good χ^2 for the precision electroweak fit. Of course, when $\lambda_5>\lambda_1>0$, V is globally unstable with respect to large (CP-violating) field directions, $\phi_1^0=ix_1$ and $\phi_2^0=x_2$ (x_1 and x_2 real) with $x^2\equiv x_1^2+x_2^2\to\infty$ and $4\lambda_5 x_1^2x_2^2/x^4>\lambda_1$. However, it has been argued [8] that such globally bad directions are not likely to have been entered during the evolution of the universe following the big bang so long as the local ($m_{A^0}^2>0$) stable minimum is nearer to the origin in field value space than the globally unstable regions. Thus, for $m_{A^0}>50\,\text{GeV}$ the global instability of the potential form that minimizes our precision electroweak χ^2 should probably not be regarded as a problem.

To achieve $\alpha\sim 0$ and $m_{H^\pm}\sim m_{H^0}\sim m_{h^0}\gg m_{A^0}$, one should choose $\lambda_1=\lambda_2=\lambda_3=\lambda_5=-\lambda_4$, i.e. the $\lambda_1=\lambda_5$ limit of the above form (which has only a globally flat limit). Small deviations from this form would then allow for the $m_{H^\pm} > m_{H^0} \gtrsim m_{h^0}$ configuration required for minimum χ^2 when $\alpha \sim 0$. Finally, if $\alpha = \beta$ we have $m_{h^0}^2 = m_{A^0}^2 + v^2\lambda_5$ and $m_{H^0}^2 = v^2\lambda_1$. Then, $m_{H^\pm} \sim m_{A^0} \sim m_{H^0} \gg m_{h^0}$ provided $\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_5 = -\lambda_4 = m_{A^0}^2/v^2$, yielding a potential form with no global problems.

6. Higher energy LC and LHC. Having delineated the dangerous choices of parameters for which failure to detect a Higgs boson at a $\sqrt{s} \leq 800\,\mathrm{GeV}$ e^+e^- collider can be rather consistent with precision electroweak constraints, we wish to address the question of whether increased LC energy or LHC running *would* allow Higgs discovery. First, by comparing the $\sqrt{s} = 500\,\mathrm{GeV}$ and $\sqrt{s} = 800\,\mathrm{GeV}$ no-discovery wedges, we see that although the $\tan\beta$ extent of the wedge narrows considerably with increasing $\sqrt{s}$, the smallest no-discovery value of m_h increases rather slowly; thus, one cannot absolutely rely on h detection at higher LC energy in these scenarios. Also, the absence of ZZ coupling and the moderate value of $\tan\beta$ implies that the h will not be detectable at the LHC. Consider next the H. Since $\chi^2_{\rm min}$ is always achieved for m_H at the $Hb\overline{b}$ threshold and for masses of the other Higgs bosons often much larger than m_H, the discovery possibilities for the $H = h^0$ (H^0) deserve particular attention in the $h = A^0$ (h^0) cases.

Consider first the cases where $h = A^0(h^0)$ and $\Delta\chi^2_{\rm min}$ is achieved when the $H = h^0(H^0)$ is SM-like. For the $\Delta\chi^2_{\rm min}$ values of m_H and for a substantial range above, the LHC would detect the H in the gold plated $ZZ \to 4\ell$ channel. If $\sqrt{s}$ of the e^+e^- collider is pushed beyond 1 TeV, and the H is not seen in ZH or $\nu\overline{\nu}H$, the minimum m_H would move into the $\gtrsim 1\,\mathrm{TeV}$ range for which one would expect to see strong WW scattering behavior at both the LHC and the LC. Our results indicate that precision electroweak fits do not necessarily have particularly bad χ^2 for such large m_H — typically, $\Delta\chi^2_{\rm min}$ only increases by $< 1-2$ compared to values obtained for $m_H \sim 800\,\mathrm{GeV}$.[♯4] As a result, only an LC with $\sqrt{s}$ sufficiently large to probe a strongly interacting WW sector could be certain of seeing a signal for the H. Note that although the $b\overline{b}H$ production channel opens up as the $\sqrt{s}$ of the LC is increased, $\sigma(b\overline{b}H)$ for a SM-like H is very small at high mass and $b\overline{b}H$ production would not be detectable.

Continuing to focus on the $h = A^0$, $H = h^0$ SM-like with $\sin^2(\beta-\alpha) \sim 1$ situation, the two heaviest Higgs bosons H^0 and $H^\pm$ have fairly large masses (e.g. $600 - 800\,\mathrm{GeV}$ at a $\sqrt{s} = 500\,\mathrm{GeV}$ LC) for the best χ^2 values. Although the $Z \to A^0H^0$ coupling would have full strength, the masses are such that A^0H^0 production would become kinematically allowed only with a substantial increase in $\sqrt{s}$. Because of the small cross sections for Yukawa processes at moderate $\tan\beta$, much larger $\sqrt{s}$ would be needed for $b\overline{b}H^0$ and $b\overline{t}H^+ + \overline{b}tH^-$ production.

[♯4]This conclusion is not entirely iron-clad for the following reason. The heavy Higgs masses are given by $\sqrt{\lambda}v$, where λ is some combination of the λ_i. To obtain masses of order $1\,\mathrm{TeV}$ (as typical for good χ^2 in the $\sqrt{s} = 800\,\mathrm{GeV}$ wedge), we need $\sqrt{\lambda} \sim 4$ or $\lambda^2/(4\pi) \sim 1$. Thus, our perturbative precision electroweak computations undoubtedly begin to break down for m_H values much above $800 - 900\,\mathrm{GeV}$.

And, much larger $\sqrt{s}$ would also be required for H^+H^- and $t\bar{t}H^0$ production. For $\sqrt{s} = 800\,\mathrm{GeV}$, the $\Delta\chi^2_{\rm min}$ cases with $\sin^2(\beta-\alpha) \sim 1$ have m_{H^0} and $m_{H^\pm}$ above 1 TeV. A $\sqrt{s} > 2\,\mathrm{TeV}$ LC would be needed to discover these Higgs in pair production. Because of the moderate value of $\tan\beta$, this would also be true for Yukawa processes. For moderate $\tan\beta$ and masses as large as discussed above, H^0 and $H^\pm$ detection at the LHC would not be possible due to the smallness of the ZZH^0 and WWH^0 couplings and the very modest size of $b\bar{b}H^0$ production. Thus, in these $\Delta\chi^2_{\rm min}$ cases, the first focus should be on LHC observation of the h^0 as a resonance or in strong WW scattering.

Next consider the $h = A^0$ cases for which $\Delta\chi^2_{\rm min}$ is achieved for small $\sin^2(\beta-\alpha)$, as typified by the moderate $\tan\beta$, $\alpha \sim 0$ cases. The $H = h^0$ will be hard to detect in the SM-like discovery modes. However, A^0h^0 production would have more or less full strength coupling and observation would be possible when kinematically allowed. Since our searches were performed subject to the requirement $\sqrt{s} < m_{h^0} + 10\,\mathrm{GeV}$, this would typically require very substantially larger $\sqrt{s}$ than the assumed value. However, in these cases the H^0 has SM-like ZZ, WW coupling and m_{H^0} is usually not much larger than m_{h^0} (which is always $\sqrt{s} - 10\,\mathrm{GeV}$). Consequently, H^0 detection in the gold-plated modes at the LHC or at a $\sqrt{s} \gtrsim 1\,\mathrm{TeV}$ LC would be possible.

Finally, for the $h = h^0, H = H^0$ (with H^0 SM-like) case, the two heaviest Higgs bosons are the A^0 and $H^\pm$. As $\sqrt{s}$ at the LC is increased, h^0A^0 production would become kinematically allowed (and be full strength), followed by H^+H^- pair production. For the moderate $\tan\beta$ values in question, the Yukawa processes would not be useful (either at the LC or the LHC).

7. Conclusions. Although it is likely that a light SM-like Higgs boson exists and will be found at the LHC and LC if not in the near future at LEP2 or the Tevatron, it is advisable to consider scenarios in which the Higgs boson(s) with substantial WW/ZZ coupling are heavy. We have shown that even the very excellent level of precision achieved for electroweak observables is not adequate to clearly prefer a Higgs sector (either 1HDM or 2HDM) with a light SM-like Higgs over a type-II 2HDM model with parameters chosen so that no Higgs boson will be discovered at a $\sqrt{s} = 500 - 800\,\mathrm{GeV}$ machine in $e^+e^- \to Zh$, $\nu\bar{\nu}h$, $b\bar{b}h$ or $t\bar{t}h$. In particular, we have considered the case of the CP-conserving 2HDM model with a light pseudoscalar $h = A^0$ or scalar $h = h^0$ with zero ZZ/WW coupling and all other Higgs bosons too heavy to be produced. For moderate $\tan\beta$, there is a range of m_h for which the h will typically not be detectable even with very high integrated luminosity and yet the masses of the other Higgs bosons can be chosen to be $\gtrsim \sqrt{s}$ *and* so that the precision electroweak fit is within $\Delta\chi^2 = 1 - 2$ of the best SM fit. Although some tuning of parameters is required for the smallest $\Delta\chi^2$, the parameter relations required are small perturbations relative to an interestingly simple form of the Higgs potential. Fortunately, Giga-Z electroweak precision measurements will allow much stronger discrimination between the light SM-like Higgs and the 'no-e^+e^--discovery' 2HDM scenarios, while $\gamma\gamma$ collisions will allow h discovery for $m_h < 0.8\sqrt{s}$ for some portion of the low to moderate $\tan\beta$ regions of the no-discovery wedges, depending upon the effective $\gamma\gamma$ integrated luminosity.

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